An Investigation into the Elastic Constants of Rocks, More Especially with Reference to Cubic Compressibility

BY

FRANK D. ADAMS AND ERNEST G. COKER

WASHINGTON, D. C.:
Published by the Carnegie Institution of Washington
June, 1906.

CARNEGIE INSTITUTION OF WASHINGTON

PUBLICATION No. 46

PRESS OF GIBSON BROS.

WASHINGTON, D. C.

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INTRODUCTION.

The question as to the amount of cubic compression which rocks may undergo under the stresses to which they are subjected in the earth's crust is one which has a direct bearing on many very important problems in geophysics. It is, however, a subject which has been but little investigated as the experimental difficulties connected with it are very considerable. The importance of a series of determinations of the cubic compressibility of a few typical plutonic igneous rocks was some time since impressed upon the authors by Mr. G. K. Gilbert, with a request that if possible they should make such determinations in connection with the researches on rock deformation which are now being carried out at McGill University under the auspices of the Carnegie Institution of Washington. An examination of all the direct methods proposed or adopted for the measurement of the cubic compressibility of solids showed that none of these could be satisfactorily applied to such materials as rocks, but the indirect methods based on Hook's law and which have been applied to metals and other compact isotropic bodies having an approximately perfect elasticity promised to give satisfactory results if applied to certain rocks, more especially to the class of rocks referred to above, viz, the acid and basic plutonic rocks, which form the greater part at least of the outer portions of the earth's crust. The present paper sets forth the methods adopted and the results obtained.

The work which was carried out in the laboratories of McGill University was commenced by the authors whose names appear on the title page, and was carried well towards completion when Dr. Coker was called to take the professorship of mechanical engineering in the Finsbury Technical Institute of London, England. He was accordingly obliged to give up the work of the research and his place was taken by Mr. Charles McKergow, lecturer in mechanical engineering in McGill University, but who immediately on the completion of the work was appointed to the professorship in mechanical engineering in the University of Virginia. A large number of the very careful measurements of elastic constants which are given in the paper were made by the latter gentleman.

METHODS WHICH MAY BE USED IN THE DETERMINATION OF THE ELASTIC CONSTANTS OF MATERIALS.

The determination of the cubic compressibility of solid substances is, as above mentioned, beset with serious difficulties. On the one hand, every direct method which has been suggested presents experimental difficulties which tend to impair its accuracy, while on the other hand the indirect methods are based on assumptions as to the isotropy of the materials, which are not warranted in the case of certain rocks. The indirect methods depending on the theory of elasticity are capable of considerable variation, and it is of interest to examine them in some detail in order to see whether certain of them at least may not be depended upon to give reliable and satisfactory results.

The determination of the elastic constants of metals has engaged the attention of many physicists and at the present time a large amount of information exists as to the values of these constants for various metals.

It is well known that in homogeneous elastic substances a simple compression stress causes a lateral strain, which bears a fixed ratio to the compression strain for any particular substance within the limit of elasticity. If, then,* we call p_x the stress on a plane perpendicular to x in the direction x, and e_x the corresponding strain, then for a direct compression stress p_x there will be a strain in the direction of this stress of amount p_x/E, where E is Young's modulus, and lateral strain of magnitude p_x/mE, where m is the ratio of the longitudinal compression to the lateral extension per unit of length.

If we suppose further that a body is subjected to cubical stress of intensity p_x, we easily see that for small and therefore superposable strains the cubical strain e_c is

$$e_c = 3p_x \frac{m-2}{mE}$$

and since the modulus of cubical compressibility D is the ratio of the stress per unit of area to the cubical strain produced, we have

$$D = \frac{p_x}{e_c} = \frac{1}{3}\frac{m}{m-2}E.$$

Hence if we know E and m we can calculate the value of D.

Further, it is shown in treatises on elasticity that if C is the modulus of shear, then

$$C = \frac{1}{2}\frac{m}{m+1}E$$

*See Ewing's Strength of Materials, Chapters I & II.

and since C and E are quantities which can be ascertained by experiment, we can from them calculate m and D.

In an important paper by Nagaoka* this latter method has been used to determine the elastic constants of a series of rocks. The value of E was determined by supporting a bar at the ends and measuring the angular change at the support due to a given load applied at the center; the value of E is then obtained by the formula $E = 3wl^2 / 4bd^3\theta$, where l is the length of the bar between the supports, b is the breadth of the bar, d the depth, and θ the angular change at the ends for a load, W. In order to determine the value of m, a specimen of rectangular section was twisted by a given torque, T, and the amount of the strain measured. It has been shown by St. Venant that for such a case the value of C is given by the formula

$$T = C\theta b^3 h \left[\frac{16}{3} - \frac{32^2 b^4}{\pi^5} \sum_{0}^{\infty} \frac{\tan h\,(2n+1)\dfrac{\pi h}{2b}}{(2n+1)^5} \right]$$

where θ is the angular change, and from this formula values of C were calculated from the observations.

This method appears to us to be open to some minor objections in that the formula for determining E is based upon a theory of flexure, which although sufficient for many purposes is nevertheless only approximate, and it is well known that values of E obtained by flexure experiments in this manner often differ from the values of E obtained by direct compression experiments by not inconsiderable amounts.

Further, in experiments upon the deflection of beams cut from rocks, it is difficult to obtain consistent readings, because of the time effect of the loading, and this difficulty is noticed in the paper cited.

As an example of the results obtained in this way, we may quote the results of certain experiments made by us with a pure white marble from Vermont.

Lath-shaped pieces of the marble were carefully prepared and were suspended on two wedge-shaped supports and then loaded in the middle. The weights were placed in a light brass pan, hanging from a thick wire which passed over the middle of the lath and lay flat upon it.

Each experiment occupied about half an hour, and the deflection was measured by attaching a scale to the marble and reading it with reference to a thin wire stretched in front of the specimen, a properly mounted telescope being employed for this purpose. The marble was in all cases placed so that its broader surface rested on the terminal supports.

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*Elastic Constants of Rocks and the Velocity of Seismic Waves. H. Nagaoka. Phil. Mag., Vol. L, 1900, p. 53.

Of the several experiments made two may be selected. The pan and wire in each case weighed 3 ounces.

In the first experiment the marble had the following dimensions: Length, 12 inches; length between supports, 11 inches; breadth, 1.259 inches; thickness, 0.284 to 0.298 inch.

The figures obtained are as follows:

	Inch.
Load with pan only (taken as zero point)	0.486
with pan plus 4 ounces	.487
8 ounces	.488
12 ounces	.489
16 ounces	.490
20 ounces	.491
24 ounces	.491
28 ounces	.492
32 ounces	.493
36 ounces	.494
40 ounces	.497
44 ounces	.498
48 ounces	.500
52 ounces	.501
56 ounces	.503
60 ounces	.505
60 ounces (after 2 minutes)	.506
64 ounces	.515
64 ounces (after 1½ minutes)	.516
66 ounces	.517
68 ounces	.518
68 ounces (after 1½ minutes)	.520
70 ounces	.521
72 ounces	.522
72 ounces (after 1 minute)	.522
74 ounces	.526
76 ounces	.528
76 ounces (after 1½ minutes)	.531
78 ounces	.533
80 ounces	.534
80 ounces (after 2 minutes, moving fast)	.540
82 ounces	.541
82 ounces (after 1½ minutes)	.543
72 ounces (weight reduced, large permanent set)	.542
84 ounces	.547
86 ounces	.549
86 ounces (after ½ minute; broke)	.554
Total deflection before breaking	.064

In the second experiment the marble lath was longer and at the same time somewhat thicker. Its dimensions were as follows: Length, 16 inches; length between supports, 15 inches; breadth, 1.229 to 1.284 inches; thickness, .347 to .356 inch.

	Inch.
Load with pan only	.343
with pan plus 8 ounces	.349
16 ounces	.368
24 ounces	.389
24 ounces (after 1½ minutes)	.392
28 ounces	.401
32 ounces	.416
32 ounces (after 1½ minutes)	.423
36 ounces	.438
40 ounces	.460
Load with pan only (weight removed, large permanent set)	.412
with pan plus 40 ounces (after 2 minutes)	.471
44 ounces	.492
44 ounces (after a few seconds)	.500
44 ounces (after 1 minute; broke)	.520
Total deflection	.177

Here it will be noticed that when a certain load is reached a distinct movement sets in and is maintained without any further increase of load, the movement growing in amount as the limit of the strength of the rock is approached and producing a permanent set.

Experiments on the determination of the elastic constants of rocks when subjected to twist were also found to be frequently unsatisfactory, owing to the low ultimate shearing values of many rocks.

While a glance at the list of rocks whose elastic constants have been measured by Nagaoka will at once show that most of them are rocks whose elasticity must be of a very imperfect kind, *e. g.*, weathered clay slate, Schalstein, tuff, etc.; the method which he has employed for the determination of Young's modulus gives very low results, even in the case of rocks such as marble and granite, where the elasticity might be supposed to be of a high order, and comparable to that which these rocks have been shown to possess in the case of the types selected for investigation in the present paper. This is shown by the following figures comprising the values obtained by him for each of the marbles and granites contained in his list.

Paleozoic marble:	*E* (Young's modulus).
No. 11A	10,120,000
11B	7,950,000
12A	5,440,000
12B	4,770,000

Granite:	*E* (Young's modulus).
No. 69 (Shodoshima)	6,140,000
68 (Hitachi)	2,853,000
71 (Hitachi)	2,175,000
56 (Hitachi)	1,588,000
52 (Hitachi)	3,265,000

Of these marbles No. 11, if a mean of the two readings be taken, has about the same modulus as the average of those on our list, while No. 12 is very much lower. The highest value given for any granite in Nagaoka's list, viz, No. 69, is somewhat higher than that of the lowest of the granites in our series, that from Stanstead. The other granites examined by Nagaoka have values for E assigned to them which are so low that they are comparable only to that of the sandstone in our series. Of the three sandstones included in Nagaoka's list the Izumi sandstone of the Mesozoic has modulus of 1,322,000, while the other two, which belong to the Diluvium, have values for E of 587,500 and 583,000, respectively.

And so when an attempt is made to calculate the cubic compression D from the values given in Nagaoka's list and obtained by his method, it is found that a negative value is actually obtained in about one-third of the rocks which he has examined. His figures, however, were intended chiefly for the purpose of calculating the velocity of the propagation of earthquake shocks.

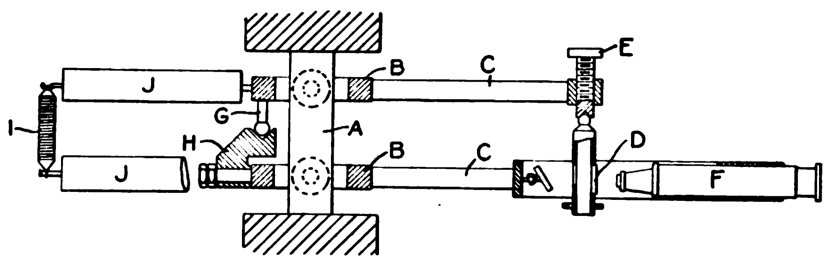


FIG. 1.—Instrument for determining the modulus of a simple strain.

In consequence of the somewhat unsatisfactory results obtained in our preliminary experiments with this method, as well as the facts with regard to Nagaoka's figures just mentioned, it was decided to adopt a somewhat different method and one which avoided both torsion and flexure and depended simply on strain produced by simple compressive stress. This will be termed the "method of simple compression."

Among the possible indirect methods, this seems to be the most satisfactory, as the assumptions necessary in the calculation of compressibility are reduced to a minimum, and the range of stress for which the ratio of stress to strain is practically constant is great. We were able to measure the strains obtained very accurately, by means of an apparatus forming part of the equipment of the testing laboratory of McGill University, for the use of which we are indebted to Dean Bovey.

This is an instrument designed by Professor Ewing, and of which a diagrammatic representation is given in figure 1, in which A is a specimen of the rock

gripped by screws passing through a pair of collars, *B*, which are 1.25 inch apart, to which latter metal rods, *C*, are attached. The upper rod carries a glass plate, *D*, with a fine line scratched upon it, the position of which can be adjusted by a screw, *E*, while the lower rod carries a micrometer microscope, *F*. The upper and lower collars, *B*, are connected by a stud, *G*, the upper one engaging with the conical hole of the swivel piece *H* in the lower, and contact is maintained by a spring, *I*, while the weights of the microscope and projecting arms are balanced by lead cylinders, *J*. A buzzer was attached to the upper lead cylinder which, when operated, caused a slight vibration in the instrument, producing a perfect adjustment as the pressure was applied.

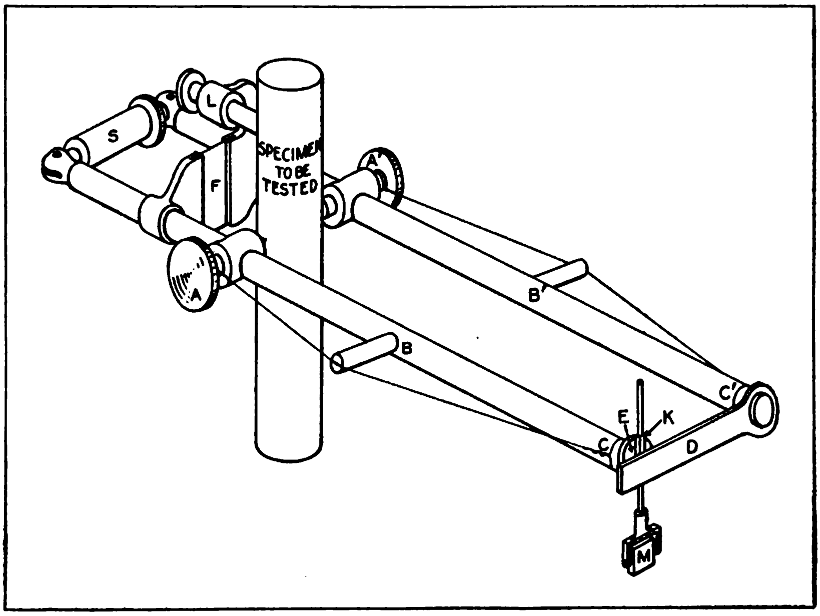


FIG. 2.—Perspective view of lateral extensometer.

The proportions of this instrument were so adjusted that one division on the micrometer scale corresponded to $\frac{1}{250,000}$ of an inch, and before using it the instrument was calibrated by aid of a Whitworth measuring machine and was found to be in correct adjustment. This instrument enabled us to determine the modulus of simple compression with great accuracy.

The linear strain perpendicular to the length of the specimen was measured by an instrument which had been designed by E. G. Coker some time previously for experiments on the lateral strains developed in metals.* Figure 2 is

*See Proceedings Royal Soc., Edinburgh, Session 1904–5. Vol. xxv, pt. vi.

a diagrammatic view of the apparatus, which consists of a pair of brass tubes, B, B', provided with set screws, A, A', for attachment to the specimen, and connected together by a flexible steel plate, F, forming the fulcrum. The ends of the tubes near the fulcrum plate are pressed apart by an adjustable spring S, to insure a uniform pressure on the screw points gripping the specimen. On the opposite end of the tubes is a spring finger, D, of ebony, pressing against a double knife-edge, K, seated in a shallow V notch cut in the end of the other arm. The knife-edge carries an adjustable mirror, M, so that if any change in the diameter of the specimen occurs the two tubes move relatively to one another in a horizontal plane and thereby cause the knife-edge mirror to rotate; the rotation of this latter is observed and measured by a telescope and scale placed at a suitable distance.

For convenience in adjustment there is a screw, L, for tilting the apparatus about the axis of the gripping screws, and the tubes B, B' are trussed to prevent vibration. This instrument was calibrated by aid of a Whitworth measuring machine and the scale adjusted so that one division corresponded to one-millionth of an inch.

APPLICATION OF THE METHOD OF SIMPLE COMPRESSION TO THE DETERMINATION OF THE CUBIC COMPRESSIBILITY OF METALS.

The behavior of such metals as wrought iron and steel over a wide range of stress shows that these metals may be considered as almost perfectly elastic. The results of the theory of elastic bodies may therefore be applied in their cases with great confidence.

As a typical example of the behavior of such materials we may consider the deportment of a specimen of wrought iron when subjected to a cycle of compression stresses, commencing at 1,000 pounds and rising to 9,000 pounds, afterwards returning to the original load.

The readings obtained for the longitudinal and lateral strains will show in such a case that equal increments or decrements of load produce strains which are very exactly proportional thereto. This is clearly shown in a plot of these readings, where the ordinates represent the total load and the abscissæ represent strains. In both cases the relation of stress to strain is represented by a straight line returning upon itself. Traces which vary but little from the ideal straight line are given by black Belgian marble, as will be seen on page 25.

Such results afford an arbitrary standard by which can be judged the degree of approximation to perfect elasticity exhibited by other metals and by rocks under similar conditions.

If we now calculate the value of the modulus E for simple compression, since this is the relation of the compression stress p to the strain e, we have

$$p = Ee$$

If we call A the cross-sectional area of the specimen when stressed by a load, P, and x the decrease of length over a measured length, L, gripped between the screw points of the measuring apparatus, we obtain

$$E = \frac{PL}{xA}$$

which, in case of a specimen of wrought iron examined for a range of 8,000 pounds, gave a value of 28,100,000, the units being pounds and inches.

The ratio m of the longitudinal strain to the lateral strain in the same case was 3.65, and using the formula

$$D = \frac{1}{3}\frac{m}{m-2}E$$

we obtain for the modulus of cubical compression (or bulk modulus) D, the value 21,300,000, a constant for the material, the reciprocal of which gives the decrease in volume of 1 cubic inch for 1 pound of pressure.

While certain rocks, such as many of the marbles, have a structure identical with that of wrought iron, most of the rocks constituting the earth's crust are composed of several minerals, and thus resemble cast iron in character, the gray variety of this substance being an aggregate of crystals or individuals of the metal iron (wrought iron), graphite, etc.

It will therefore be of interest to ascertain how a specimen of cast iron behaves under compression stress, and how far its elasticity falls short of that which would be exhibited by a perfectly elastic body.

For this purpose a fine-grained specimen of somewhat hard cast iron was faced and tested. The results of this test are given in the following table, and the stress-strain curves are plotted in figure 3. I represents longitudinal compression and II lateral extension.

The behavior of cast iron, as exhibited by these experimental results, shows a falling away from the theoretical standard of perfect elasticity, but even in the most perfectly elastic bodies there is probably a slight hysteresis effect, so that we are justified in using the results obtained to calculate the modulus of compressibility, if the error introduced thereby is negligible or very small.

It may be pointed out that this method and others of the indirect type have been freely used to obtain values of the bulk modulus for cast iron and metals of like character, and it will be shown that the composite crystalline rocks are very similar to cast iron in their behavior under stress, although generally more perfectly elastic.

Cast Iron.

Size	1.034 × 1.006		1.034	1.006
Area	1.041	1.041	....	....
E	15,000,000	15,000,000	....	....
σ	.25	.25	....	....
D	10,000,000	10,000,000	....	....
C	6,000,000	6,000,000	....	....
Longitudinal compression (multiply readings by 4 for millionths).			Lateral extension—(millionths).	
Load (in pounds).	Side *P.*	Side *U.*	Side *P.*	Side *U.*
1,000	0	0	0	0
2,000	19	20	12	11
3,000	40	37	26	21
4,000	60	58	41	32
5,000	80	78	56	48
6,000	100	100	72	65
7,000	120	120	86	83
8,000	140	143	102	99
9,000	160	160	119	116
8,000	145	143	106	108
7,000	123	125	90	85
6,000	104	110	76	70
5,000	85	90	60	60
4,000	63	63	44	50
3,000	44	40	30	39
2,000	20	21	13	21
1,000	0	0	6	9

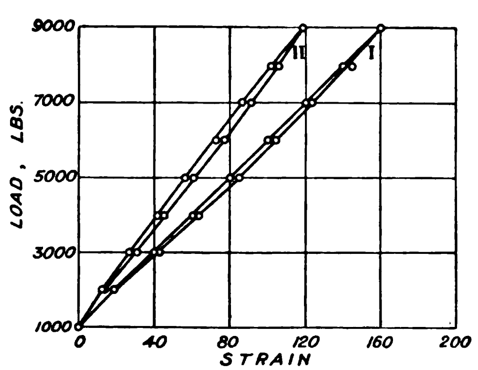


FIG. 3.—Cast iron. Stress-strain curves.

APPLICATION OF THE METHOD OF SIMPLE COMPRESSION TO THE DETERMINATION OF THE COMPRESSIBILITY OF ROCKS.

It has been noted in the case of marble when subjected to bending stress that the strain as exhibited by the deflection of a point of the bar increases with the time, and the strength under shear produced by a torque was also found to be so small that a determination of the strain was very difficult to measure. These difficulties have been noted by Nagaoka,* who states that:

> Preliminary experiments on granite show that Hooke's law does not hold even for very small flexure and tension, and that the after effect is very considerable from the pressure, when the prism is sufficiently loaded or twisted, the deviation from the direct proportionality between strain and stress was incomparably great as compared with that observed in metal. This must be chiefly due to the low limit of elasticity, so that it is necessary to experiment only within very narrow limits of loading and twisting. These limits are widely different for different specimens of rocks, and the modulus of elasticity, as well as that of rigidity, was always determined with such stresses as will approximately produce strains proportional to them. The deviation from Hooke's law was prominent in certain specimens of sandstone, and it was more marked in tension than in flexure experiments; in certain rocks it is indeed doubtful if anything like a proportionality between stress and strain can be found, even for extremely small change of shape. On releasing these rocks from stress the return toward the former state is extremely small, showing that the elasticity of the rock is of a very inferior order.

These observations of Nagaoka for bending and twisting have been confirmed by our own deflection experiments, as above mentioned.

If, however, the rock be subjected to direct compression, strains in which the time effect is small and the lag of the strain is also small are almost invariably obtained. This is especially the case if before the actual experiment is carried out the material be several times subjected to a range of stresses at least as great as those employed in the experiment itself. This preliminary stressing brings the material to "a state of ease," and is also commonly adopted when the elastic constants of metals are determined.

It is evident, therefore, that this direct compression method may with confidence be applied to the measurement of the cubic compression of rocks, although as mentioned below the accuracy of the result so obtained will differ with different classes of rocks.

If the rock be massive, compact, and crystalline (or glassy) the method can be safely employed and good results will be obtained. If, on the other hand, the rock is schistose, porous, or loosely coherent, the method will from the nature of the case be very much less satisfactory.

The plutonic igneous rocks as a class most nearly resemble the metals in structure, being holocrystalline and massive, and therefore present the

*Elastic Constants of Rocks and the Velocity of Seismic Waves. H. Nagaoka. Phil. Mag., vol. L, 1900, p. 58.

nearest approach among rocks to perfect elasticity; they are therefore a class of rocks to which this method is especially applicable. It fortunately happens that they are also a class of rocks a knowledge of whose compressibility is of special importance for the elucidation of many geological problems, constituting as they do the greater part of the earth's crust.

A second class of rocks which are comparable with them in their approach to perfect elasticity comprises the marbles and certain limestones.

A series of sixteen typical rocks representative of these two classes were accordingly selected for measurement. Under the first class a number of granites were chosen as representing the acid plutonic rocks and a number of types of the gabbro-essexite series were selected as representing the basic plutonic rocks. In all these cases great care was taken to choose the most homogeneous and massive rocks of each series and to secure test pieces free from all flaws and cracks. As representing the second class a number of typical marbles and limestones, also perfectly massive in character, were selected. For purposes of comparison, or contrast, a sandstone was added to the list as being a rock which, on account of its more or less porous nature could hardly be expected to yield satisfactory results by this method.

An examination of the stress-strain curves of these 16 rocks, omitting the sandstone, shows that on the average they possess a rather more perfect elasticity and exhibit less hysteresis than cast iron. Some of them, as for instance the Baveno granite, the nepheline syenite, the diabase, and the black Belgian marble, show much better curves, approximating in fact to the straight lines given by wrought iron, which may be considered for our present purpose as expressing perfect elasticity.

The close approximation to perfect elasticity is shown by the return of the curve to its initial or starting point, and the amount of the hysteresis is shown by the width of the loop.

The width of this hysteresis (or lag) curve or loop, indicates the amount of the divergence from Hook's law which the material exhibits—this law being that the stress and strain are *directly* proportional. When the curve is narrow, as it is in all cases except the Stanstead granite and the sandstone, the divergence from Hook's law is not great enough to seriously affect the result.

The rocks, therefore, with these exceptions, fulfil the conditions of elasticity necessary to the successful application of the method. In these two cases the results are less certain, owing to the greater hysteresis of the rock.

It might at first sight appear that while the method employed is theoretically perfect as applied to the measurement of the compressibility of vitreous rocks and of very fine grained crystalline rocks, a considerable error might be introduced when the rocks are coarser in grain. In the case of all the common crystalline rocks, the individual grains of which the rock is composed

are anisotropic, that is, they have different moduli of elasticity in different directions. In massive rocks such as those investigated, however, these grains occur in the rock with an absolutely irregular orientation and would in the case of a fine-grained rock mutually compensate for one another in any transverse line along which the expansion of the rock under compression might be measured. If, however, the rock were coarser in grain, fewer individual crystals would be found in any transverse line of section, and there might possibly in this way be a lack of compensation, as the rock in one section might be composed of grains whose axis of greater elasticity approximated on an average more nearly to the direction of measurement than in other sections. If such were really the case, there should be in these coarser-grained rocks an exceptionally great variation in the readings obtained from different specimens of the same rock, as well as from the different sections in the same specimen.

But such is not the case, as will be seen by an examination of the figures in accompanying table. They represent the results obtained from ten measurements of the compressibility of Baveno granite, which is coarse in grain, and ten of Sudbury diabase, which is very fine in grain, together with eight measurements on Tennessee limestone, which is rather coarse grain, and seven on plate glass. They were made in each case on two or more specimens cut from the same mass and the measurements of the expansion were made on several different planes through each, so that in every case the measurement was effected in a different line through the rock, all of these, however, of course being at right angles to the direction of the compressive stress and lying in the medial plane of the column.

Full details concerning each measurement will be found in the tables which set forth the results obtained, under the sections dealing with the several rocks in question. The size of grain and the texture of the rock can also be seen by examining the photomicrographs and color prints of the polished surfaces of the respective rocks.

	Max.	Min.	Diff.
Baveno granite (coarse) 10 trials	4,880,000	4,380,000	500,000
Sudbury diabase (very fine) 10 trials....	11,170,000	9,655,000	1,515,000
Plate glass, 13 trials....................	6,930,000	6,020,000	910,000
Tennessee marble (rather coarse) 7 trials .	6,130,000	5,770,000	360,000

It will thus be seen that there is no correspondence between the coarseness of grain and the magnitude of the variations in the readings obtained. The differences in glass, which is an isotropic material in which the elasticity is equal in all directions, are greater than in the Tennessee marble, which is rather coarse in grain, and in Baveno granite, which is the coarsest rock of

the set. The greatest differences obtained are those found in the finest grained rock in the series, viz, the Sudbury diabase.

It is evident, therefore, that the different moduli of elasticity of the constituent grains of a rock do not introduce any perceptible error in measurements made by this method, when a column an inch in diameter is employed, and when the rocks are not coarser in grain than the Baveno granite. In fact, while surrounded on all sides by other grains, no individual grain can expand freely, as it would if subjected to compression unhampered by any surrounding medium, and thus the anisotropic character of the individual grains produces but little effect on the elasticity of the rock as a whole.

These experiments also show that in the case of rocks composed of several minerals it makes no perceptible difference whether the points of attachment of the instrument are embedded in the grains of one mineral or of another.

The chief source of error and the one to which the variations observed are for the most part to be attributed seems to be a mechanical one, viz, the difficulty of getting an ideal contact between these points of attachment and the specimen to be measured, especially in view of the extremely small dimensions of the movement to be measured.

The question of the influence of temperature on the elasticity and compressibility of rocks is of course one which has an important bearing on certain problems of geophysics. The only investigation of this subject, so far as can be ascertained, consists of a few preliminary experiments by Nagaoka and Kasakabe.* In these the torsion method was employed, and the experiments were carried out on a single rock, viz, sandstone. This rock, as has already been mentioned, being porous and stratified in character, is a material whose elastic properties are far from ideal. The results are summed up by the authors in the following words:

> Preliminary experiments with sandstone show that the modulus of elasticity is much affected by the variation of temperature, *i. e.*, about 0.5 per cent per degree. It does not, however, necessarily diminish with the increase of temperature where the temperature is low, *i. e.*, it is maximum about 9° C.

As has been shown however, the values for the elastic constants obtained by this torsion and bending method have yielded results which can not in all cases be correct and which differ very considerably from those obtained by the more direct and simple method which has been employed in the present paper. These results bearing on the variation of elasticity induced by changes of temperature, especially in view of the fact that they are stated by the

*Modulus of Elasticity of Rocks and Velocities of Seismic Waves. Publications of the Earthquake Investigation Committee, No. 17. Tokyo, 1904, p. 43.

investigators to be "preliminary," can as yet hardly be taken as of general application to all rocks, even if correct for the specimen of sandstone examined.

In our own investigations the laboratory was maintained at a temperature of from 63° to 68° F. (17.2° to 20° C.), and a thorough investigation into the effect of temperature was not undertaken, as this would be very difficult to carry out when employing the method of direct compression used, the difficulty consisting in heating the specimen itself without in any way affecting the measuring apparatus attached to it.

It seemed, however, possible to ascertain whether any serious change in the elastic constants of the massive crystalline rocks employed in the present investigation would result from a moderate change of temperature. For purpose of trial the rock selected was the Sudbury diabase, a typical fine-grained plutonic rock. A column of it was placed by Mr. McKergow in a small testing machine having a capacity of 50 tons, and the temperature of the room in which the machine was set up having been lowered to + 10° F., a cycle of compression readings were taken in the usual way adopted when Young's modulus is to be determined. The temperature of the room was then raised by about 10° and another cycle of readings were taken. It was then raised another 10° and a third series of readings were obtained, and so through successive stages of 10° until the normal temperature of the room (about 65° F.) was reached. The initial reading of the instrument before the application of pressure was of course different in each case, owing to the expansion of the rock which followed from heating. These initial points were plotted on a line, and the results obtained when the specimen was subjected to a certain maximum load, together with the increase of temperature at each stage, were plotted on a second line. If the compression was greater at 65° than at 10° for the same load these two lines should have diverged, but as a matter of fact they were practically parallel. The differences between the readings given by the same load at different temperatures were no greater than those obtained by different measurements under the same load at the same temperature. The conclusion therefore seems to be indicated that a change of temperature made no perceptible difference within the range of temperatures employed, although a difference of 0.5 per cent for each degree centigrade, which was Nagaoka's result, would mean a difference of about 25 per cent in range of temperature employed by Mr. McKergow.

While, therefore, this experiment can not be considered as supplying accurate information concerning the effect produced by a rise in temperature on the elastic constants of rocks, for the instruments themselves are in some measure affected by the same changes of temperature, they serve to show that in the case of the massive crystalline rocks the influence of temperature is probably not very great. The subject is one which requires further investigation.

THE METHOD OF MEASUREMENT.

In carrying out the measurements, prisms of the rock approximately 1 inch square and 3 inches long were usually employed (see fig. 4). These were cut and ground with smooth faces, but were not polished. In these two small round holes were drilled in the medial line of each vertical face for the purpose of attaching the instrument, when Young's modulus was to be measured. These holes were made by means of a small diamond drill and were perfectly round and smooth. They were each 0.05 to 0.08 inch in diameter and 0.125 inch deep and 1.25 inches apart, lying at equal distances above and below the center of the prisms. These holes were champfered at the outer end, as shown in figure 4, and were found to afford the most perfect attachment which could be secured for the points of the instrument. By means of these prisms two sets of measurements of the vertical compression could be made with each prism, by attaching the instrument first to one pair of opposite faces and then to the other.

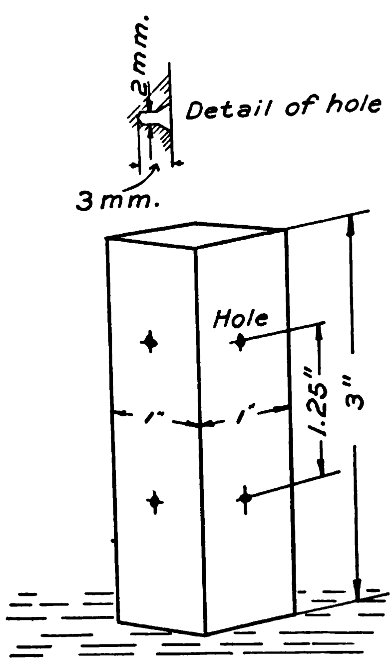


FIG. 4.—Square test specimen of rock.

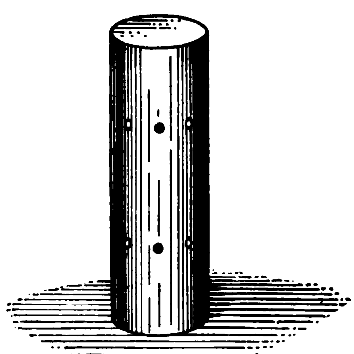


FIG. 5.—Round test specimen, showing position of holes.

In some cases round columns were used (see fig. 5). These were approximately 1 inch in diameter and 3 inches in length. With these it was possible to make four sets of measurements in compression with each column, by drilling eight pairs of holes, as above described, whose planes intersected at angles of 45° instead of 90° as in the square prisms.

It was of course necessary in every case, whether prisms or columns were employed, to exercise great care to have the ends of the test pieces very

carefully faced and absolutely parallel to one another. Before the actual measurements were made, the rock in every case was brought to a "state of ease" in the manner already described.

The pressure was applied in most cases by a 100-ton Wickstead testing machine, which was so carefully adjusted that it was sensitive to a load of 4 pounds.

The specimen, having been placed in the press and reduced to a state of ease, was then after careful adjustment submitted to loads increasing in successive stages of 1,000 pounds until the limit of safety had been reached, when the load was reduced successively by the same amounts, accurate readings being taken at each increment and decrement of load. The maximum load employed in the case of most rocks was 9,000 pounds, equivalent to from 9,000 pounds to about 11,500 pounds per square inch, according to whether a square or round prism was employed. In the case, however, of some of the stronger rocks a load of as much as 15,000 pounds per square inch was employed.

In the determination of the lateral strain, which was made upon the same set of columns as those used for measuring the vertical compression, care was taken that the theoretical conditions were realized, and that the material was free to expand laterally, as otherwise the values obtained for the lateral extension would be inaccurate. In all cases, therefore, the measuring apparatus was set as nearly as possible upon the central section of the test piece, and the ends of the specimen, after being ground smooth, were coated with a thin film of oil, so that the polished pressure plates of the machine would have as little tendency as possible to prevent freedom of lateral expansion.

In a number of cases accurate measurements were taken during the successive cycles of loading and unloading to which the specimen was subjected in order to bring it to a state of rest. These are recorded in the case of the Baveno granite and the Stanstead granite and served to show how the hysteresis of the rock may be reduced to a minimum by subjecting the test piece to this process. The measurements of each cycle usually occupied from 10 to 15 minutes.

It was at first conjectured that in the case of rocks composed of several minerals differences of reading might result from the attachment of the extensometer to different portions of the rock, the points of the instrument being fixed in some cases in grains of one mineral and in other cases in grains of another. It was found, however, as has already been mentioned, that the measurements on two sets of prism faces made in the manner above described, or on the four planes intersecting the vertical columns, where these had been provided with eight pairs of holes, showed that in the case of the rocks examined the differences between the several measurements on the same prism seem to be unaffected by the circumstance above referred

to. The differences between the measurements thus made on rocks composed of several minerals were no greater than those found in the case of the limestones, which were composed of the single mineral calcite, or on glass.

In the case of the majority of the rocks investigated, a number of prisms or columns cut from the same block of rock were measured in order to ascertain whether different test pieces would give identical readings. It was found as a result of these investigations that the differences between the different specimens were no greater than those which were obtained by measuring the same specimens with the instrument attached at different places. In the case, however, of the Quincy granite, test pieces from two different blocks of the rock were prepared, and it was found that while the several measurements made on each test piece agreed among themselves, there was a distinct divergence in the elastic constants of the two specimens of the rock. This was probably due to a difference in composition, as the two rocks differed somewhat in appearance.

In the case of the green gabbro from New Glasgow, the results obtained by measurements made upon different parts of the same prism were discordant, for reasons which will be pointed out and which were dependent upon the structure of the rock.

Fifty-five specimens of rock, nineteen of glass, and two of iron were employed in this investigation and every conceivable precaution was taken to insure the attainment of accurate results. The rocks in all cases were air dry, having been allowed to remain in the laboratory for several weeks after they had been cut, before the measurements were made.

In the accompanying tables the following elastic constants are given:

E = Young's modulus, *i.e.*, the quotient of the longitudinal stress by the longitudinal compression.

σ = Poisson's Ratio; this is the reciprocal of m.

D = Modulus of Cubic Compression $= \frac{1}{3}\left(\frac{m}{m-2}\right)E$. The reciprocal of this gives the decrease in volume of a cubic inch of the material for a pressure of 1 pound per square inch applied on every side.

C = Modulus of Shear $= \frac{1}{2}\left(\frac{m}{m+1}\right)E$, which is the quotient of torsional stress to torsional strain.

m = The ratio of longitudinal compression to lateral extension per unit of length.

E and m are measured directly; the other values are calculated from them.

These values in the case of each rock are given in the respective tables, expressed in inch and pound units, and the results are summarized in a general table on page 69.

The measurements were made in these units on account of the fact of the testing machine employed was graduated to read pounds.

For purposes of comparison, however, this latter table has been recalculated in C. G. S. units, and the results are set forth in the second table to be found on page 69.

In the case of metal, Poisson's ratio is generally arrived at by stretching the bar and determining the value of the longitudinal extension divided by the lateral contraction. In case of rocks the tensile strength being low and the materials being brittle, it is more convenient and more accurate to make the determination by compressing a short bar or column, and determining the value of the longitudinal compression divided by the lateral expansion. This gives the value designated as m, of which Poisson's ratio is the reciprocal. Theoretically one method is as accurate as the other. In actual practice it might be supposed that the short compression columns in question would not expand quite so much at the ends as in the middle because of the friction against the compression plates. In order, however, to cause these to slip as easily as possible over the ends of the column, the surface of the rock in contact with them was always made very smooth and also was slightly oiled. It was found that, these precautions being observed, the expansion at the ends of the column was practically as great as at the center, where the measurement was taken, the differences being so small that no serious discrepancy was introduced.

In the tables the first transverse line designates the specimen employed as a, b, c, or d. The second line gives the diameter of the specimen, which is often slightly different in the two directions. The length of the column in all cases was about 3 inches, but this is not stated in the table, as the compression is not measured on the total length of the column, but on the length of that portion of it which lies between the points of attachment of the instrument.

The third line gives the area, which is approximately 1 square inch in the case of a square prism and three-quarters of a square inch in the case of a round column.

In the four succeeding lines the four elastic constants E, σ, D, and C, are given, as determined by each measurement.

Another transverse line contains the letters U or P, which designate the two diameters of the column when two measurements were made on the same square prism, these two directions being always at right angles to one another. In the case of round columns, on which measurements were frequently made in several planes, these are designated as "first holes," "second holes," etc.

In each table there follows the values obtained for successive loadings of 1,000 pounds in the case of each specimen, first for compression, when the figures multiplied by four give millionths of an inch, and then for lateral expansion, given directly in millionths of an inch. These afford the data for calculating the constants and for plotting the curves which accompany every table.

In the figures for the constants of iron and of one or two of the rocks, which are the result of measurements which were made at the beginning of the investigation, a slight correction has been made, owing to the inaccurate calibration of the extensometer, which will explain a certain discrepancy which will appear if the figures are recalculated.

THE ELASTIC CONSTANTS OF ROCKS COMPOSED OF A SINGLE MINERAL.

MARBLES AND LIMESTONES.

BLACK BELGIAN MARBLE, BELGIUM.

This rock is known in trade by the name of "Belgian black" or "Noir fin." It is an extremely fine grained black marble which takes a very high polish and is used very extensively in interior decoration. It has a splintery fracture, breaking almost like glass.

When thin sections are examined under the microscope the rock is found to be so fine in grain that a high power is necessary to resolve it. It is composed of minute calcite grains from 0.02 mm. to 0.002 mm. in diameter and of irregular shape, between and around which are occasional minute films and spots of a black color.

In this very fine grained and even groundmass are embedded a very few larger forms of clear white calcite, some of them rodlike, others circular in shape, and others possessing more complicated outlines. These are evidently of organic origin, representating small fragments of fossils. They are very sparsely scattered through the rock. The rock also contains occasional minute grains or crystals of iron pyrites.

Fragments of this rock dissolve readily in cold dilute hydrochloric acid, giving off a fetid odor and leaving a considerable amount of a light flocculent residue, black in color and apparently consisting of some form of bituminous matter. In the residue there are also a few minute grains of pyrite.

Plate I A is a color-process photograph of a polished surface of this marble and Plate I B is a photomicrograph of a thin section of the rock, taken in ordinary light and magnified 27 diameters.

A square prism of the rock of the usual dimensions was employed to measure the elastic constants, and the results are set forth in the table found on page 25.

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 27 DIAM.-ORDINARY LIGHT)

TRENTON LIMESTONE, MONTREAL, CANADA.

Olivine Diabase, Sudbury, Province of Ontario, Canada—Continued.

No.	*a*	*a*	*b*	*b*	*c*	*c*
Size.....	.981	.981	.983	.983	.983	.983
Area ...	.756	.756	.758	.758	.758	.758
E.......	13,250,000	13,780,000	14,020,000	14,320,000	14,020,000	14,320,000
σ.......	.2865	.281	.291	.277	.291	.283
D	10,340,000	10,460,000	11,170,000	10,720,000	11,170,000	11,000,000
C	5,160,000	5,380,000	5,430,000	5,620,000	5,430,000	5,580,000

LONGITUDINAL COMPRESSION—MULTIPLY READINGS BY 4 FOR MILLIONTHS.

Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*
1,000..	0	0	0	0	0	0
2,000..	30	30	30	25	30	30
3,000..	60	60	60	55	60	60
4,000..	90	90	90	95	90	85
5,000..	125	120	115	110	120	115
6,000..	155	150	145	140	150	145
7,000..	185	180	175	170	180	175
8,000..	225	215	210	200	210	205
9,000..	250	240	235	230	235	230
8,000..	220	210	210	200	210	205
7,000..	190	185	170	175	180	175
6,000..	165	155	145	140	150	145
5,000..	130	125	115	115	120	115
4,000..	105	100	85	95	90	85
3,000..	75	60	55	55	60	60
2,000..	45	25	30	25	30	30
1,000..	15	0	0	0	0	0

LATERAL EXTENSION—MILLIONTHS.

No......	*a*	*a*	*b*	*b*	*c*	*c*
Size.....	.981	.981	.983	.983	.983	.983
Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*
1,000..	0	0	0	0	0	0
2,000..	28	28	28	25	21	27
3,000..	54	51	54	49	49	53
4,000..	83	78	82	73	78	79
5,000..	111	103	110	100	105	107
6,000..	140	130	138	122	131	130
7,000..	169	156	164	149	160	154
8,000..	198	183	191	172	185	183
9,000..	225	210	215	200	215	205
8,000..	200	185	192	174	195	180
7,000..	172	155	170	150	170	166
6,000..	144	135	140	125	145	130
5,000..	115	110	118	100	115	110
4,000..	85	78	88	75	85	80
3,000..	52	54	56	50	55	55
2,000..	22	26	30	25	25	26
1,000..	0	0	0	0	0	0

As will be seen, the values obtained for D in this rock are considerably higher than those yielded by any other rock of the series examined. In the six independent measurements carried out on the first three specimens, the difference between the highest and lowest values for D amounted to 830,000 pounds, while on the four measurements made on specimen *d* there is a rather greater difference amounting to 845,000 pounds.

The averages of the determinations made on each of these columns are as follows:

	E	D	σ	C
a	13,515,000	10,400,000	0.2838	5,270,000
b	14,170,000	10,945,000	0.2840	5,525,000
c	14,170,000	11,085,000	0.2870	5,505,000
d	13,197,750	10,076,000	0.2812	5,155,000
Average......	13,763,187	10,626,500	0.2840	5,363,750

The stress-strain curves given by a specimen this rock are shown in figure 23. As will be seen from these curves, in its approach to perfect elasticity the rock is comparable to plate glass.

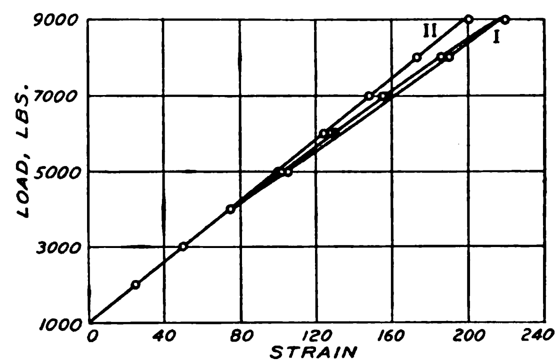


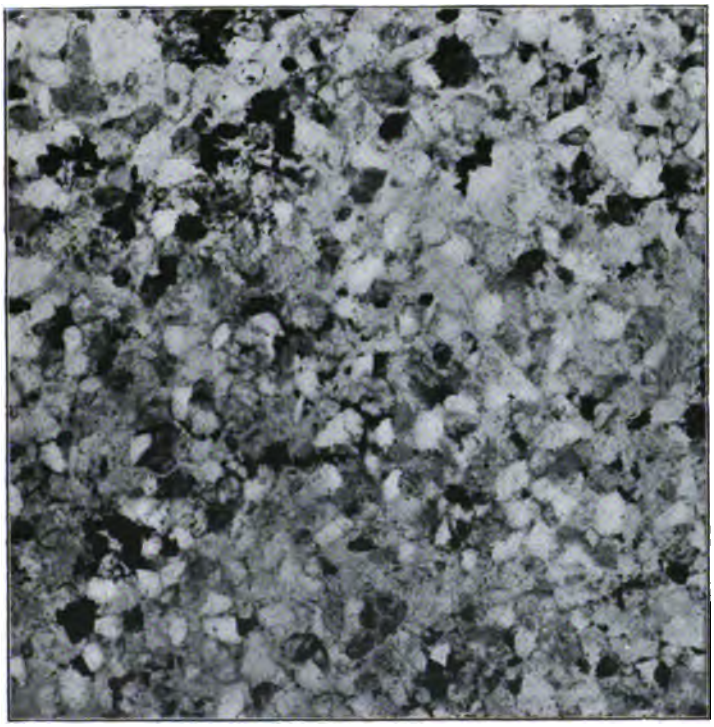
FIG. 23.—Sudbury Diabase. Stress-strain curves.

SANDSTONE, CLEVELAND, OHIO, UNITED STATES.

This is a fine and even grained yellowish sandstone used very extensively for building purposes. The bedding is marked by a slight variation in color in different beds. The prism of the rock used in determining its elastic constants was cut from a single bed of uniform character and color, and was taken in the plane of the bedding. A color-process photograph of a smooth surface of the rock is shown in Plate XVI A.

Under the microscope it is seen to be a typical highly feldspathic sandstone. The constituent minerals are present in grains which are approximately

A. PHOTOGRAPH OF FLAT SURFACE, (NATURAL SIZE)



B. PHOTOMICROGRAPH OF THIN SECTION, (X 27 DIAM.-ORDINARY LIGHT)

SANDSTONE, CLEVELAND, OHIO.

uniform in size and of rudely rounded or subangular outline. The quartz grains are clear and fresh; the feldspar individuals, which are abundant, on the other hand, are for the most part in an advanced stage of alteration, being always turbid and in most cases quite opaque, from the presence of alteration products. Some few grains of comparatively unaltered plagioclase are, however, present, and scattered through the rock there is a considerable amount of hydrated oxide of iron, which often lies between the grains, forming a cement. The rock, however, also contains a not inconsiderable amount of calcite, which causes it to effervesce slightly when treated with dilute hydrochloric acid, and which is also seen to lie between the clastic grains also forming a cement, often in the form of individuals of a size comparable to those of the other minerals.

The rock, however, is not a crystalline rock, but a typical clastic one. There is not a continuous crystalline web or mosaic, but a mass of rounded or subangular grains which are in part cemented together as above described, but in part are separated by minute open spaces. It is to be expected, therefore, that the rock will show serious defects in elasticity, as proves to be the case when attempt is made to determine its elastic constants. A photomicrograph of the rock taken in ordinary light and multiplied 27 diameters is shown in Plate XVI B.

A square prism of the rock was employed, and it was found to be dangerous to submit it to a load of over 4,000 pounds, the crushing weight of the rock being much lower than that of the other rocks, which are crystalline in texture. The figures obtained are given in the following table:

Sandstone, Cleveland, Ohio, United States.

Size	1.000 × 1.025	1.000
Area	1.025	....
E	2,290,000	....
σ	.29	....
D	1,816,000	....
C	888,000	....
Load (in pounds).	Longitudinal compression (multiply readings by 4 for millionths).	Lateral extension (millionths).
1,000	0	0
2,000	152	110
3,000	288	241
4,000	426	396
3,000	309	305
2,000	175	178
1,000	4	0

The stress-strain curves are shown in figure 24.

As will be seen, the rock displays a marked hysteresis and is not therefore an ideal material for the application of this method of determining compressibility.

The results obtained are as follows:

$$E = 2{,}290{,}000; \quad \sigma = 0.29; \quad D = 1{,}816{,}000; \quad C = 888{,}000.$$

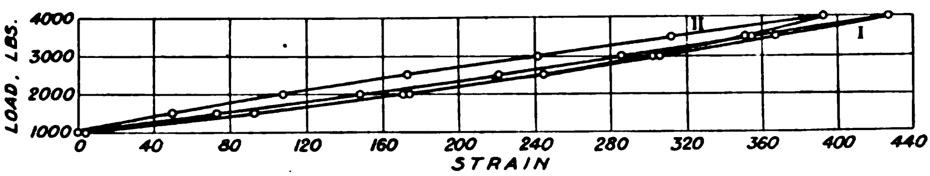


FIG. 24.—Sandstone. Stress-strain curves.

THE ELASTIC CONSTANTS OF GLASS.

As in geophysical speculations, the earth in respect to its rigidity and compressibility is often compared to a globe of glass, it seemed advisable to determine as accurately as possible the elastic constants of glass, for the purpose of comparing them with the results obtained in the case of the various rocks considered in this paper, employing the same methods and carrying out the work under exactly the same conditions. This material lends itself excellently to this method of measuring these constants, provided the glass is free from all irregularities in its substance and is isotropic in character. The first difficulty experienced was that of obtaining such a glass. At the outset it was thought that thick glass rods such as are used for various purposes in the chemical and physical laboratory might be employed, but although several lots of the purest variety of this material were procured, the glass constituting it was found in all cases to contain minute air bubbles, and when examined between crossed nicols in polarized light, showed brilliant colors—red, yellow, and blue. This indicated a state of marked tension in the glass, evidently due to the rod having been drawn when the glass was in a viscous state, which was also shown by the circular arrangement of the little bubbles in the rod, following the direction of its surface. Short lengths of this rod, moreover, when tested in compression, so soon as the maximum load had been exceeded, instead of splitting from top to bottom, broke as if composed of a series of rudely concentric shells. All attempts on the part of the various glass makers to whom this glass was submitted for a thorough annealing, failed to remove or in fact to reduce to any considerable extent this anisotropic condition.

The figures obtained from one of these glass rods approximately an inch in diameter are given in the following table:

Glass Rod.

Size	.985	.97
Area	.774	....
E	8,075,000	....
σ	.2	....
D	4,485,000	....
C	3,361,000	....
Load (in pounds).	Longitudinal compression (multiply readings by 4 for millionths).	Lateral extension (millionths).
1,000	0	0
2,000	55	32
3,000	95	61
4,000	145	95
5,000	200	123
6,000	250	155
7,000	300	....
8,000	350	....
9,000	....	....
8,000	350	....
7,000	305	....
6,000	250	155
5,000	205	125
4,000	160	100
3,000	115	65
2,000	65	35
1,000	5	2

That the tension in this glass seriously affected the results obtained—as might be expected—is clearly seen in the value for D being much too low, as will be shown later.

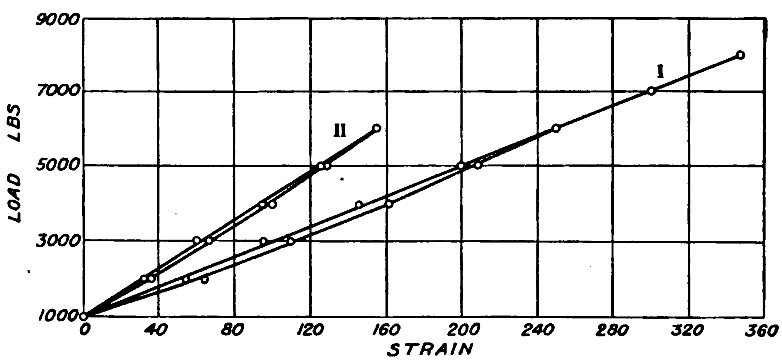


FIG. 25. Glass Rod. Stress-strain curves.

The stress-strain curves plotted from these values are shown in figure 25. As will be seen, the material exhibits a distinct hysteresis.

After a prolonged search for isotropic glass in masses of sufficient size to measure the elastic constants, it was found that plate glass answered the requirements. A piece of one-inch plate glass made in Great Britain was accordingly secured and was cut into strips an inch wide, and these again into three-inch lengths. The square prisms thus produced were then properly faced and polished. The glass was found to be absolutely free from all flaws and impurities and when examined between crossed nicols the prisms, although an inch thick, showed in one direction at right angles to vertical axis absolute blackness throughout a complete revolution, while in the other direction at right angles to this there was during a revolution an alternation of blackness with a pale grayish illumination. This change was so slight that, considering the thickness of the glass and the sensitiveness of the test, the material may

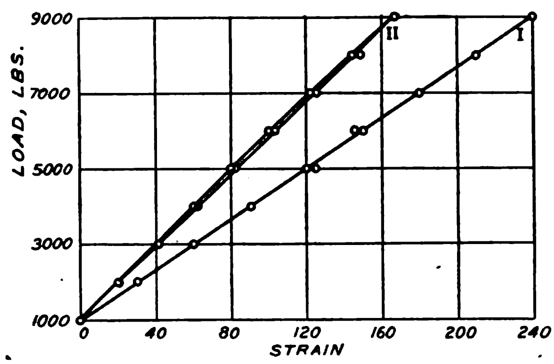


FIG. 26.—Plate Glass. Stress-strain curves.

be considered to be practically free from internal tension and to be isotropic in character.

In order to get a good average and to eliminate chance errors as far as possible, seven of these prisms were taken, and two complete sets of determinations were made on each of them, using in every case different pairs of faces. Fourteen determinations were thus made of each of the elastic constants. The figures obtained are set forth in the table on page 65.

In this table a complete series of values obtained from each specimen are given in double rows. When the average of all these results is taken, the values obtained for the several constants of plate glass are as follows:

$$E = 10{,}500{,}000; \quad \sigma = 0.2273; \quad D = 6{,}448{,}000; \quad C = 4{,}290{,}000.$$

The stress-strain curves given by one of the prisms is shown in figure 26. In this figure I represents longitudinal compression and II lateral extension.

Plate Glass.

No. . .	*a*	*b*	*c*	*d*	*e*	*f*	*g*
Size. . .	.9855×1.0205	.9865×1.0055	.981×1.0135	1.016×1.008	1.0215×.9955	1.022×1.0025	1.025×.994
Area. .	1.0057	.992	.994	1.024	1.017	1.024	1.016
E.	10,350,000 10,590,000	10,950,000 10,500,000	10,480,000 10,350,000	10,380,000 10,380,000	10,450,000 10,930,000	10,380,000 10,600,000	10,450,000 10,230,000
σ. . . .	.2281 .228	.236 .2341	.226 .235	.233 .227	.221 .23	.229 .225	.216 .215
D.	6,370,000 6,480,000	6,930,000 6,580,000	6,460,000 6,520,000	6,480,000 6,350,000	6,380,000 6,760,000	6,370,000 6,430,000	6,140,000 6,020,000
C.	4,220,000 4,310,000	4,440,000 4,250 000	4,280,000 4,190,000	4,210,000 4,230,000	4,280,000 4,440,000	4,220,000 4,330,000	4,300,000 4,360,000

LONGITUDINAL COMPRESSION—MULTIPLY READINGS BY 4 FOR MILLIONTHS.

Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*
1,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2,000	30	30	25	30	30	30	30	30	30	30	28	25	30	30
3,000	60	55	55	60	60	60	60	60	60	55	55	55	60	60
4,000	90	85	85	90	90	90	90	90	90	85	85	85	85	90
5,000	120	115	115	120	120	120	120	115	115	110	115	110	115	125
6,000	145	145	145	145	150	150	150	145	145	135	145	140	145	145
7,000	175	175	175	175	180	180	180	175	175	170	175	170	180	175
8,000	210	205	205	210	210	210	205	205	205	195	200	195	205	210
9,000	240	235	230	240	240	243	235	235	235	225	235	230	235	240
8,000	210	205	205	210	210	210	205	205	205	195	205	195	205	210
7,000	180	180	175	180	180	180	180	175	175	170	175	170	175	180
6,000	150	145	145	150	150	155	150	145	145	135	145	140	145	150
5,000	120	115	115	120	120	120	120	115	115	110	115	115	115	120
4,000	90	85	85	90	90	90	90	90	85	85	85	85	85	90
3,000	60	55	55	60	60	60	60	55	55	60	60	60	60	60
2,000	30	30	25	30	30	30	30	30	30	30	30	30	30	30
1,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0

LATERAL EXTENSION—MILLIONTHS.

1,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2,000	22	22	21	22	19	22	20	19	19	19	20	19	21	20
3,000	44	45	42	45	38	46	42	39	39	40	41	39	42	42
4,000	68	68	66	68	60	69	63	60	60	59	65	60	60	62
5,000	89	89	88	89	81	92	86	81	82	80	89	79	80	83
6,000	111	111	109	111	103	115	112	105	102	100	109	100	100	103
7,000	132	133	129	133	124	138	134	126	124	122	130	122	122	122
8,000	153	156	151	156	149	160	157	148	145	143	155	145	144	144
9,000	173	175	172	179	170	180	178	169	169	165	176	165	167	165
8,000	155	155	152	157	149	163	159	149	149	145	155	147	149	147
7,000	135	133	130	135	125	140	135	129	126	125	130	125	125	125
6,000	115	112	110	115	103	115	100	106	105	105	112	105	103	106
5,000	93	90	90	90	80	90	89	85	88	38	90	85	82	89
4,000	70	71	70	70	60	65	64	62	62	62	69	65	62	69
3,000	49	45	45	50	40	42	40	42	41	41	45	43	42	49
2,000	22	23	22	25	21	20	20	21	21	21	22	22	22	28
1,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Determinations of the cubic compressibility of glass, D, have been made by other observers using various methods. The results go to show that different varieties of glass vary considerably in their compressibility. These determinations may be tabulated as follows:*

Everett	5,074,600 to 6,379,400 (C. G. S.$=3.5$ to 4.4×10^{11}).
Amagat—Common glass	6,745,000 (.000002181 per atmosphere).
Amagat—Crystal glass	6,112,300 (.000002405 per atmosphere).
Tait	5,657,700 (.0000026 per atmosphere).

As will be seen, the figures obtained for plate glass in the present investigation lie a little above the average of the various values here given, and are nearly those of the highest value obtained by Everett.

SUMMARY OF RESULTS.

The table on page 69 gives a summary of the average values obtained for E, σ, C and D in the case of all rocks examined in this investigation. With these are placed, for purposes of comparison, the results obtained for these constants in the case of wrought iron, cast iron and glass. In the second table on page 69 these values are again presented, recalculated into C. G. S. units.

The rocks fall naturally into three groups, differing from one another in compressibility, but the several members of each group agreeing fairly closely among themselves.

These three groups show a corresponding difference in composition.

The first group consists of the marbles and limestones. These have an average value for D of 6,345,000. One of these, however, the Black Belgian marble which is very much finer in grain than the others and breaks almost like a piece of glass, has a very much higher value for D than that possessed by the other rocks which among themselves are nearly identical. If we omit this Belgian marble, the average of D for the other limestones and marbles, is 5,855,000.

The second group comprises the granites. These again show a close agreement of values among themselves, except in the case of the Stanstead granite, which rock, as already mentioned, shows a defective elasticity. The average value of D for the granites is 4,399,000.

The third group embraces the basic intrusives (gabbro, anorthosite, essexite, and diabase). These show greater differences, but have an average value for D of 8,825,000. The nepheline syenite, although higher in silica and therefore properly speaking an acid rock; in its freedom from quartz, and its richness in feldspar (although the feldspar is largely orthoclase instead of plagioclase), in mineralogical composition belongs with these basic rocks rather than with the granites. It also approaches the essexite most nearly in its compressibility.

*See Everett, Illustrations of the C. G. S. System of Units with tables of Physical Constants. MacMillan & Co., 1902, pp. 60 to 64. The figures there expressed in various units have been here recalculated into inch-pound values.

If the nepheline syenite be included with the basic rocks, an average value of D is obtained of 8,308,000.

This omits from consideration the sandstone, it being a rock of an entirely different class from the others, and furthermore one which shows so much hysteresis that the application of this method to it is less satisfactory than in the case of the other rocks of the series.

These results may be presented as follows:

	Average of D.
Marbles and limestone	6,345,000
Granites	4,399,000
Basic intrusives	8,308,000

The cause of the much greater compressibility of granite as compared with the marbles and basic intrusives is not clear, but would seem to be connected with the presence of quartz. The only determination of the cubic compressibility of quartz, so far as can be ascertained, is one by Voigt,* the value obtained being 5,504,190 pounds (387×10^6 grams per sq. cm.). This compressibility, as will be seen, is much greater than that found in the case of either the limestones or the basic intrusives, and while not in itself sufficiently great to account for the high compressibility of the granites, goes to show that in the quartz we have a mineral which is more compressible than the ordinary rock making minerals which form the chief constituents in the rocks of the series examined.

The marbles and the limestones of the earth's crust are confined to its most superficial portion, resulting as they do from the process of sedimentation. There is every reason to believe, however, that what we may term the substructure of the earth's crust is composed of acid and basic plutonic igneous rocks. These make up the lowest part of the crust to which we have access and are found coming up from the still greater depths.

The cubic compressibility D of the earth's crust must lie between the values given above for the granites and the basic intrusives, approaching one or other of these values according to the relative proportion in it of one or other of these classes of rocks.

If we take the average of the values obtained from these two classes of rocks as represented by the seven granites and the five basic intrusives (including the nepheline syenite) the values obtained for D of 6,353,500.

This, as will be seen, differs but little from the value of D obtained for plate glass which is 6,448,000.

If, therefore, the earth's crust be composed of granite and basic igneous rocks in approximately equal proportions, its compressibility will be that of glass. If it be composed almost exclusively of granite, the earth's crust will be more

*Quoted in Becker: Experiments on Schistosity and Slaty Cleavage, Bulletin 241, U. S. Geol. Survey, p. 32.

compressible than glass, and if the basic rocks preponderate very largely it will be less compressible than this substance.

It is, however, in any case much more compressible than steel, which has a value for D of from 26,098,000 to 27,547,000 (18 to 19 $\times$ 10^{11}, C. G. S.).*

The compression to which the rocks were subjected in this investigation ranged from 6,000 to 17,340 pounds to the square inch. Most of the rocks, however, were subjected to a load of from 9,000 to 15,000 pounds per square inch, and their bulk compression was determined for these loads as maxima. Higher pressures could not be employed without running the risk of breaking the specimen and at the same time of destroying the measuring apparatus. One apparatus was in fact so destroyed.

The question arises as to whether under still higher pressures, if rupture could be avoided, the ratio of load to compression would be maintained. Judging from the deportment of much stronger substances such as steel, when similarly tested, it is inferred that this ratio of bulk compression will remain constant for very much higher pressures, or until deformation sets in and the rock begins to flow.

With regard to the accuracy of the results obtained by this method as compared with those obtainable by any method in which cubic compression is actually produced and measured, it may be observed that by far the best method of this kind hitherto suggested seems to be that proposed by Richards and Stull.† We have endeavored to make use of this method in order to obtain results for purposes of comparison with those given in the present paper but have not hitherto succeeded in overcoming certain experimental difficulties. The experimental errors in this method, though apparently small, still exist, and in applying it to rocks, which are much less compressible than the substances examined by Richards and Stull, these errors become proportionately more serious. Moreover, higher pressures than those used in the method employed in the present paper could scarcely be employed in this direct method, while difficulties dependent on the possible lack of absolute continuity in the substance of the rock and the danger of minute air-filled spaces would probably present themselves in the case of most rocks. It seems that, all things being considered, the indirect method here employed is probably as accurate as any direct method which can be used. The attempt to apply Richards and Stull's method to rocks is still being continued, however, and it is hoped that satisfactory results may be eventually obtained by its use.

*Illustrations of the C. G. S. System of Units, with Tables of Physical Constants. MacMillan & Co., 1902, p. 60.

†New Method of Determining Compressibility. Published by the Carnegie Institution of Washington, December, 1903.

Elastic Constants of Rocks.

SUMMARY OF RESULTS (AVERAGE) EXPRESSED IN INCH-POUND UNITS.

Specimen.	E	σ	C	$D=\frac{1}{3}\left(\frac{m}{m-2}\right)E$
Wrought iron............	28,100,000	0.2800	11,000,000	21,300,000
Cast iron...............	15,000,000	0.2500	6,000,000	10,000,000
Black Belgian marble....	11,070,000	0.2780	4,330,000	8,303,000
Carrara marble	8,046,000	0.2744	3,154,000	5,946,000
Vermont marble.........	7,592,000	0.2630	3,000,000	5,341,000
Tennessee marble........	9,006,000	0.2513	3,607,000	5,967,000
Montreal limestone	9,205,000	0.2522	3,636,000	6,167,500
Baveno granite..........	6,833,000	0.2528	2,724,800	4,604,000
Peterhead granite	8,295,000	0.2112	3,399,000	4,792,000
Lily Lake granite........	8,165,000	0.1982	3,380,000	4,517,500
Westerly granite	7,394,500	0.2195	3,019,700	4,397,500
Quincy granite (1)	6,747,000	0.2152	2,781,600	3,984,000
Quincy granite (2)	8,247,500	0.1977	3,445,000	4,555,000
Stanstead granite........	5,685,000	0.2585	2,258,700	3,940,000
Nepheline syenite	9,137,500	0.2560	3,635,000	6,237,500
New Glasgow anorthosite	11,960,000	0.2620	4,750,000	8,368,000
Mount Johnson essexite..	9,746,000	0.2583	3,872,600	6,750,000
New Glasgow gabbro* ...	15,650,000	0.2192	6,365,000	9,555,000
Sudbury diabase	13,763,000	0.2840	5,364,000	10,626,500
Ohio sandstone..........	2,290,000	0.2900	888,000	1,816,000
Plate glass.............	10,500,000	0.2273	4,290,000	6,448,000

SUMMARY OF RESULTS (AVERAGE) EXPRESSED IN C. G. S. UNITS.

Specimen.	E	σ	C	D
Wrought iron...........	19.37×10^{11}	0.2800	7.590×10^{11}	14.680×10^{11}
Cast iron..............	10.34×10^{11}	0.2500	4.132×10^{11}	6.897×10^{11}
Black Belgian marble....	7.24×10^{11}	0.2780	2.982×10^{11}	5.736×10^{11}
Carrara marble	5.54×10^{11}	0.2744	2.171×10^{11}	4.090×10^{11}
Vermont marble.........	5.24×10^{11}	0.2630	2.069×10^{11}	3.680×10^{11}
Tennessee marble........	6.21×10^{11}	0.2513	2.482×10^{11}	4.115×10^{11}
Montreal limestone	6.35×10^{11}	0.2522	2.504×10^{11}	4.250×10^{11}
Baveno granite..........	4.71×10^{11}	0.2528	1.875×10^{11}	3.179×10^{11}
Peterhead granite	5.71×10^{11}	0.2112	2.340×10^{11}	3.300×10^{11}
Lily Lake granite........	5.63×10^{11}	0.1982	2.330×10^{11}	3.103×10^{11}
Westerly granite	5.09×10^{11}	0.2195	2.080×10^{11}	3.029×10^{11}
Quincy granite (1)	4.64×10^{11}	0.2152	1.916×10^{11}	2.750×10^{11}
Quincy granite (2)	5.68×10^{11}	0.1977	2.373×10^{11}	3.140×10^{11}
Stanstead granite........	3.92×10^{11}	0.2585	1.556×10^{11}	2.718×10^{11}
Nepheline syenite	6.29×10^{11}	0.2560	2.505×10^{11}	4.290×10^{11}
New Glasgow anorthosite	8.25×10^{11}	0.2620	3.275×10^{11}	5.760×10^{11}
Mount Johnson essexite..	6.71×10^{11}	0.2583	2.670×10^{11}	4.650×10^{11}
New Glasgow gabbro *...	10.80×10^{11}	0.2192	4.380×10^{11}	6.589×10^{11}
Sudbury diabase	9.49×10^{11}	0.2840	3.700×10^{11}	7.329×10^{11}
Ohio sandstone..........	1.58×10^{11}	0.2900	$.612 \times 10^{11}$	1.250×10^{11}
Plate glass.............	7.24×10^{11}	0.2273	2.960×10^{11}	4.439×10^{11}

*See page 57